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NOTES ON THE USE OF DEMPSTER-SHAFER AND FUZZY
REASONING TO FUSE IDENTITY ATTRIBUTE DATA

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by

D. J. Kewley

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NOTES ON THE USE OF DEMPSTER-SHAFER AND FUZZY REASONING TO FUSE IDENTITY ATTRIBUTE DATA

by

D.J. Kiley

SUMMARY

Dempster-Shafer evidential reasoning (D-S) has been applied to fusion of identity attribute data by Filippidis and Schapel. Their example involved deciding, from a given list, which radar emitter was producing the frequency and pulse rate measured by an ESM receiver. The D-S method was used to reach conclusions under uncertainty. Here an alternative approach using fuzzy reasoning is compared to their results. It is seen that the same conclusions are reached with a considerable reduction in computation. The ability to consider new data after a conclusion is reached is seen to be an additional reason for favouring the use of fuzzy reasoning for data fusion of this type. A discussion on the relationship between the approaches is provided.

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JULY 92

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1 INTRODUCTION

Recently Filippidis and Schapel [1] have described an application of Dempster-Shafer [2, 3, 4] evidential reasoning (D-S) to the problem of fusing identity attribute data for a defence application. The purpose of using D-S was to account for uncertainty in the measurements of two types of parameters from an Electronic Support Measures (ESM) receiver. By using this method, an identification of a radar emitter (aircraft or ship) can be determined. Thus an example of data fusion was given.

The purpose of this paper is to indicate that the processes of D-S are more computationally intensive and not necessarily more informative than an alternative approach using fuzzy reasoning [6, 7]. In this demonstration parts of the above paper [1] are reproduced here in a revised form. Fuzzy reasoning is then applied to the same problem. It is inferred from these results that the application of fuzzy reasoning in data fusion problems is more useful.

While comparisons between Dempster-Shafer reasoning and fuzzy reasoning have been made many times before (eg [13, 10, 12]), an additional purpose of this paper is to bring these methods to the attention of DSTO staff not currently involved with the problems of dealing with uncertainty in systems.

2 THE IDENTITY ATTRIBUTE PROBLEM

The identity attribute problem [1] is concerned with an ESM receiver being used to measure the carrier frequency (rf) and pulse width (pw) of pulsed radar signals. From these two measurements an identification of the radar emitters is required. It is known a priori that five possible emitters can be involved. These are labelled as types E1, E2, E3, E4 and E5, where the rf and pw characteristics of each type are known. Thus, in principle, it would appear that any received signal can be used to determine which emitter is involved.

However, the measurements themselves are subject to uncertainty, or errors, because the signal to noise ratio may be low. Therefore the data may have characteristics in common with several of the alternative emitters. The application of D-S and fuzzy reasoning is used to resolve the uncertainty about the choice of emitter type.

The characteristics of each of the possible five emitters are shown in Figure 1 for the carrier frequency and Figure 2 for the pulse width. These are represented by probability of each emitter having a specific rf and pw value. (Note these figures are only schematics.)

The range of signals received by the ESM receiver are shown by the vertical lines on each of the figures. The measurement errors for rf and pw are such that an uncertainty of 0.3 and 0.2 are assigned to them, respectively.

3 DEMPSTER-SHAFER EVIDENTIAL REASONING

The following discussion on D-S is essentially a revision of that given in reference [1]. Blackman [5] gives a description of this method for radar applications and his annotation is followed here.

The process of data fusion for D-S consists of finding the intersection of two sensor propositions ie the AND operation. However, since not all propositions are 100% likely, then the disjunction or union (ie the OR operation) of the propositions must be assigned to the uncertainty of the intersection. In this case, the rf and pw measurement ranges are taken as propositions about a possible emitter.

Dempster-Shafer evidential reasoning uses the concept of probability mass which for our purposes is the area under each of the emitter curves. For the ESM data of Figures 1 and 2, the masses that occur in the indicated bounds are shaded for each of the emitters. (Note they overlap each other in the figures.) These masses are then normalised so that their total mass is unity. They provide the probability for each emitter being the one making the signals that are being received. The uncertainty of the receiver is accounted for by multiplying each rf and pw mass by 0.7 and 0.8, respectively. The total mass of each proposition, including the uncertainty, is therefore unity.

In general, Shafer's basic probability assignment, m , is the function on the power set of X ($P(X)$)

$$m: P(X) \rightarrow [0,1] \text{ such that} \\ m(\emptyset) = 0 \text{ and } \sum m(A) = 1, \text{ for all sets } A \text{ in } X.$$

The probability masses found for the current example, and schematically shown in Figures 1 and 2, are represented here by the mass vectors

$$m_r = M_r = (m_r(E1), m_r(E2), m_r(E3), m_r(E4), m_r(E5), m_r(\Theta)) = (0.13, 0.22, 0.35, 0, 0, 0.3) \\ m_{pw} = M_{pw} = (m_{pw}(E1), m_{pw}(E2), m_{pw}(E3), m_{pw}(E4), m_{pw}(E5), m_{pw}(\Theta)) = (0.26, 0.08, 0.17, 0.03, 0.26, 0.2)$$

where $m_{pw}(E1) \dots m_{pw}(E5)$ are the pw mass assignments for each emitter and $m_{pw}(\Theta)$ is the uncertainty factor for pw. As the uncertainty factor is a mass assignment to all the propositions then

$$m_{pw}(\Theta) = m_{pw}(E1 \text{ or } E2 \text{ or } E3 \text{ or } E4 \text{ or } E5)$$

In D-S, the likelihood of a proposition "a" is represented by the subinterval $[Sp(a), Pl(a)]$ of the unit interval $[0,1]$. $Sp(a)$ represents the *support* for the proposition while $Pl(a)$ represent the *plausibility* of it. They represent the minimum and maximum likelihood of the proposition. They are related by $Pl(a) = 1 - Sp(\text{not } a)$, where $Sp(\text{not } a)$ is the support for all propositions that are not "a". The degree of uncertainty about the probability value for a proposition is represented by the width of the subinterval, ie $\Theta(a) = Pl(a) - Sp(a)$. For example $a(0.25, 0.85)$ gives the probability of "a" as being between 0.25 and 0.85, with an uncertainty $\Theta=0.6$. The support for a proposition is the sum of all masses assigned directly to it [5] ie

$$Sp(a) = \sum \{m(b) \mid b \subseteq a\}.$$

The plausibility is the sum of all the masses associated with the proposition and all the disjunctions, including Θ , with it, ie

$$Pl(a) = \sum \{m(b) \mid b \subset a\} \cdot 0\}.$$

The aim here is to find the combined mass, $m_3 = M_{m_1, m_2}$ by using Dempster's rule of combination. This is an extension of Bayes' rule [2, 3]. An alternative method of combining mass assignments is given by Baldwin [8], who also accounts for fuzzy descriptions of uncertainty.

The Dempster combination rules are [5]:

1. The product of mass assignments to two propositions that are consistent leads to another proposition contained within the original ie $m_1(a1).m_2(a1) = m(a1)$.
2. Multiplying the mass assignment to uncertainty by the mass assignment to any other proposition leads to a contribution to that proposition ie $m_1(\Theta).m_2(a2) = m(a2)$.
3. Multiplying uncertainty by uncertainty leads to a new assignment to uncertainty ie $m_1(\Theta).m_2(\Theta) = m(\Theta)$.
4. When inconsistency occurs between knowledge sources assign a measure of inconsistency denoted "k" to their products ie $m_1(a1).m_2(a2) = k$.

The matrix representing the combination of the mass assignments is given by the following table. Note that as the rt mass assignments are zero for E4 and E5, no columns are used.

Table 1. Combining mass vectors m_1 and m_2 using Dempster's combination rules

$m_1(\Theta)=.2$	$\Theta=.06$	$E1=.026$	$E2=.044$	$E3=.07$
$m_2(E1)=.26$	$E1=.078$	$E1=.0338$	$k=.0572$	$k=.091$
$m_2(E2)=.08$	$E2=.024$	$k=.0104$	$E2=.0176$	$k=.028$
$m_2(E3)=.17$	$E3=.051$	$k=.0221$	$k=.0374$	$E3=.0595$
$m_2(E4)=.03$	$E4=.009$	$k=.0039$	$k=.0066$	$k=.0105$
$m_2(E5)=.26$	$E5=.078$	$k=.0338$	$k=.0572$	$k=.091$
	$m_1(\Theta)=.3$	$m_1(E1)=.13$	$m_1(E2)=.22$	$m_1(E3)=.35$

The inconsistency is the sum of all the "k" terms ie $k=0.4491$. The new mass vector (m_3) of the combined proposition, is computed by summing all the corresponding entries in the matrix and normalising by the consistency $(1-k)$. Thus

$$m_3(\Theta) = 0.06/0.5509 = 0.109$$

$$m_3(E1) = (0.026 + 0.078 + 0.0338)/0.5509 = 0.250$$

$$m_3(E2) = (0.044 + 0.024 + 0.0176)/0.5509 = 0.155$$

$$m_3(E3) = (0.07 + 0.051 + 0.0595)/0.5509 = 0.328$$

$$m_3(E4) = (0.009)/0.5509 = 0.0163$$

$$m_3(E5) = (0.078)/0.5509 = 0.1416$$

and therefore $M_{m_1} = m_1 = (0.25, 0.155, 0.328, 0.0163, 0.1416, 0.109)$.

Note that as the sum of the m_1 and m_2 masses are both unity then it follows that the sum of all the cross terms on the table are also unity. As the normaliser for the computed non-"k" mass cross terms is equivalent to $1-k$ then the computation load could be reduced by not calculating the "k" cross terms at all.

Plausibility is calculated, for example, by $Pl(E1) = m(E1) + m(\Theta) = 0.25 + 0.109 = 0.359$.

Note that here all the plausibilities are just a constant shift from the support.

The mass vector suggests that emitter E3[0.328,0.437] is preferred over E1[0.25,0.359]. However, we could discern this conclusion by examination of the table entries, that is the consistency scaling did not improve our knowledge. The value of scaling comes when applying these new masses in further propositions.

Filippidis and Schapel[1] then combine this result with new information such as

$$M_{m_2} = m_2 = (0.70, 0.0, 0.0, 0.0, 0.3)$$

which indicates a high likelihood of encountering an E1 emitter. This can be used with the above combination rules to deduce a new mass vector.

Table 2. Combining mass vectors m_1 and m_2 using Dempster's combination rules

$m_1(\Theta) = .109$	$\Theta = .0327$	$E1 = .0763$
$m_1(E1) = .25$	$E1 = .075$	$E1 = .175$
$m_1(E2) = .155$	$E2 = .0465$	$k = .1085$
$m_1(E3) = .328$	$E3 = .0984$	$k = .229$
$m_1(E4) = .0163$	$E4 = .0049$	$k = .0114$
$m_1(E5) = .1416$	$E5 = .0425$	$k = .0991$
	$m_2(\Theta) = .3$	$m_2(E1) = .7$

Thus $k = 0.448$

$$m_s(\Theta) = 0.0327/0.552 = 0.059$$

$$m_s(E1) = (0.0763 + 0.075 + 0.175)/0.552 = 0.591$$

$$m_s(E2) = (0.0465)/0.552 = 0.084$$

$$m_s(E3) = (0.0984)/0.552 = 0.178$$

$$m_s(E4) = (0.0049)/0.552 = 0.008$$

$$m_s(E5) = (0.0425)/0.552 = 0.077$$

$$\text{and therefore } M_{\text{composite}} = m_s = (0.591, 0.084, 0.178, 0.008, 0.077, 0.059).$$

Again the highest mass is used to choose the new conclusion that E1 is the likely emitter, with an uncertainty of 0.06. The support and plausibility for E1 is the interval [0.591, 0.650].

It is obvious that a lot of calculation is involved in reaching a conclusion. The alternative method [8] also has the same effect. If another possible emitter was to be considered, due to other received information, then the above procedure with all the scaling effects would need to be repeated again.

Examining the calculations above, an observation about them can be made. The intersection between the propositions has the effect of keeping simultaneously large masses from each proposition while reducing the rest of the masses. In this manner only the emitter that has high masses in all the propositions is supported. If the multiplication of two masses is replaced by the minimum of them, the following table is found

Table 3. Combining mass vectors m_1 and m_2 using the minimum instead of the product

$m_1(\Theta)=.2$	$\Theta=.2$	$E1=.13$	$E2=.2$	$E3=.2$
$m_2(E1)=.26$	$E1=.26$	$E1=.13$	$k=.22$	$k=.26$
$m_2(E2)=.08$	$E2=.08$	$k=.08$	$E2=.08$	$k=.08$
$m_2(E3)=.17$	$E3=.17$	$k=.13$	$k=.17$	$E3=.17$
$m_2(E4)=.03$	$E4=.03$	$k=.03$	$k=.03$	$k=.03$
$m_2(E5)=.26$	$E5=.26$	$k=.13$	$k=.22$	$k=.26$
	$m_1(\Theta)=.3$	$m_1(E1)=.13$	$m_1(E2)=.22$	$m_1(E3)=.35$

Defining

$$m_1(\Theta) = 0.2$$

$$m_1(E1) = \min(0.26, 0.13, 0.13) = 0.13$$

$$m_1(E2) = \min(0.2, 0.08, 0.08) = 0.08$$

$$m_1(E3) = \min(0.2, 0.17, 0.17) = 0.17$$

$$m_1(E4) = \min(0, 0.03) = 0$$

$$m_1(E5) = \min(0, 0.26) = 0$$

Normalising leads to $M_{Demp} = m_1 = (0.224, 0.138, 0.293, 0, 0, 0.345)$, which is similar to the Dempster result, except that the uncertainty mass has increased.

Once again that E3 is preferred over E1, without all the computation. Using this same mass assignment method for the second stage, the following table is found.

Table 4. Combining mass vectors m_1 and m_2 using the minimum instead of the product

$m_1(\Theta) = .345$	$\Theta = .3$	$E1 = .345$
$m_1(E1) = .224$	$E1 = .224$	$E1 = .224$
$m_1(E2) = .138$	$E2 = .138$	$k = .138$
$m_1(E3) = .293$	$E3 = .293$	$k = .293$
	$m_2(\Theta) = .3$	$m_2(E1) = .7$

$m_2(\Theta) = 0.3$
 $m_2(E1) = \min(0.224, 0.345) = 0.2$
 $m_2(E2) = \min(0, 0.138) = 0$
 $m_2(E3) = \min(0, 0.293) = 0$
 $m_2(E4) = 0$
 $m_2(E5) = 0$

Therefore after normalising

$M_{\Theta, E1, E2, E3, E4, E5} = m_k = (0.4, 0.0, 0.0, 0.0, 0.6)$. This tells us that E1 is the emitter.

Clearly, in reaching a decision about which emitter is likely, the multiplication method of Dempster is not obviously superior to the minimum method suggested here. However, the uncertainty calculation is unsatisfactory due to the ad hoc nature of this approach.

The maximin criteria shown here to choose the appropriate emitter has been used elsewhere in situations of both certainty and uncertainty where multiple criteria are used in decision making [9]. It has also been shown [9] to be a similar form to fuzzy set theory.

4 FUZZY REASONING

The concept of fuzzy reasoning is to use rules combining fuzzy variables. Fuzzy variables are based upon fuzzy set theory of Zadeh [6, 7] which is used to represent uncertainty. Thus if the field of discourse X has a variable x in a fuzzy set A , then x has a membership function $\mu_A(x)$ in the unit interval $[0, 1]$. A crisp, or standard, set has membership values that are either 0 or 1. Uncertainty is represented by this membership value which varies between 0 and 1.

Operations on fuzzy sets corresponding to the logical AND, OR and NOT are defined by

$$\begin{aligned}
 A \text{ AND } B &= A \cap B = \{ (x, \min(\mu_A(x), \mu_B(x))) \}, \\
 A \text{ OR } B &= A \cup B = \{ (x, \max(\mu_A(x), \mu_B(x))) \} \text{ and} \\
 A^c &= \{ (x, 1 - \mu_A(x)) \}.
 \end{aligned}$$

More generally, it has been proposed that conjunction (AND) and disjunction (OR) can be replaced by the triangular (T) norm and the T-conorm operations, respectively [10]. Note that the D-S method uses the T-norm $(\mu_A(x) * \mu_B(x))$ whose T-conorm is $(\mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x))$. Various T norms can be defined [10] and are ranked with $\min(p, q) \geq p * q \geq \max(0, p + q - 1) \geq T_\alpha(p, q)$.

These correspond to the logical product (best case of positive correlation), the algebraic product (independent data or no correlation), the bounded product (worst case of negative correlation) and the drastic product. Thus the D-S method uses the assumption of independence of its evidence when combining them.

The fuzziness of a fuzzy set is given by the fuzzy entropy of it. As entropy measures the uncertainty of a system or message, then the fuzzy entropy represents the uncertainty of the fuzzy set. Kosko [11] has shown that the entropy is given by

$$E(A) = M(A \cap A^c) / M(A \cup A^c)$$

where $M(A)$ is the fuzzy count ($= \sum_i \mu_A(x_i)$). Thus $E(A)$ varies between certainty ($=0$) and maximum uncertainty ($=1$).

The same probability curves seen in Figure 1 can be used to represent the membership function for each emitter in the fields of discourse of rf and pw. A discussion on the relationship of probabilities and fuzziness is also given by Kosko [11] who shows that probability represents a special case of fuzziness ie fuzzy theory extends probability theory! Thus it is entirely reasonable that the membership functions for emitter E1, $E1(rf)$ and $E1(pw)$, are set to be the curves labelled E1 in both figures (similarly for the other emitters).

The fuzzy variable rf_range can be defined to have membership of $E1(rf)$ given by the area of the membership function over the bounds indicated in Figure 1. Thus the probability mass values can also be used for fuzzy reasoning. The need to scale the masses to unity is not there but can be accepted without altering the reasoning process. Multiplication of the masses by the certainty of the FSM measurement is still required and represents the "weight" of it.

For the FSM example, the fuzzy "rules" can be stated by

If rf_range is $Ent(rf)$ and pw_range is $Ent(pw)$ then the emitter is En .

where $Ent(rf)$, $Ent(pw)$ and En , $n=1..5$ represents the membership functions for each of the emitters. A standard AI (Artificial Intelligence) rule would be similar except that the answers to the "is" propositions can only be 0 or 1 (False or True) and therefore the emitter is either En or not En . For each fuzzy rule, En has a value range between, and including, 0 and 1.

Therefore applying to the identity attribute data fusion example,

$rf_range = \{ (E1,0.13), (E2,0.22), (E3,0.35), (E4,0), (E5,0) \}$ and
 $pw_range = \{ (E1,0.26), (E2,0.08), (E3,0.17), (E4,0.03), (E5,0.26) \}.$

The entropies of each fuzzy proposition are $E(rf_range) = 0.16$ and $E(pw_range) = 0.19$.

Using the fuzzy rule,

$emitter = rf_range \cap pw_range = \{ (E1,0.13), (E2,0.08), (E3,0.17), (E4,0), (E5,0) \}.$

The crisp value is the maximum > 0 so E3 is the chosen emitter with an entropy $E(emitter) = 0.08$. This entropy compares well with the D-S uncertainty mass of 0.1.

Using the prior knowledge we have

$$\text{emitter} = \text{rf_range} \cap \text{pw_range} \cap \text{prior} = \{ (E1,0.13), (E2,0), (E3,0), (E4,0), (E5,0) \}$$

with the crisp value giving E1 as the chosen emitter with an entropy of 0.026. Again the entropy compares well with the D-S uncertainty mass of 0.06.

The computation requirements are clearly less yet the identity (ID) has been successfully attributed to the same emitters as before. The entropy, or uncertainty, of the conclusion has also been shown to reduce in a consistent manner.

For the case of another emitter that may need to be considered, provided the original masses were not normalised as here, the new result can be found quickly. Note the multiplication by the certainty (or weight) is still required. Thus the new ID can be found by using

$$\text{emitter}(\text{crisp}) = \max (\text{rf_range} \cap \text{pw_range} \cap E6(\text{rf}) \cap E6(\text{pw}))$$

where $E6(\text{rf})$ and $E6(\text{pw})$ are the unnormalised weighted membership values for emitter E6 with respect to rf and pw, respectively. By contrast, the D-S method would require a recalculation of all the tables to include the new emitter.

5 DISCUSSION

5.1 Zadeh's Objection

Zadeh's objection [13] to the D-S method is the use of the normalisation to remove mass assigned to the null set. He shows that for

$$m_1(a) = 0, m_1(b) = 0.1, m_1(c) = 0.9 \text{ and } m_2(a) = 0.9, m_2(b) = 0.1, m_2(c) = 0$$

the combined result is $m_3(a) = 0, m_3(b) = 1, m_3(c) = 0$.

This result is not consistent with the low mass assigned to proposition 'b' in both probability assignments; the normalisation has concealed the dissonance (or the contradictory aspect) of the two sets of evidence. Shafer's counter example [10, 12] slightly modifies m_1 and m_2 so that

$$m_1'(a) = 0.01, m_1'(b) = 0.1, m_1'(c) = 0.89 \text{ and } m_2'(a) = 0.89, m_2'(b) = 0.1, m_2'(c) = 0.01,$$

with the new combined result of $m_3'(a) = 0.32, m_3'(b) = 0.36, m_3'(c) = 0.32$.

The lesson to be learnt from these examples is that, in some cases, there is greater danger in assignment of zero or very low values to a probability due to the normalisation procedure.

5.2 Combining additional evidence

When combining information about a particular hypothesis, it is obvious that the fuzzy reasoning AND will represent it by the one low value despite the existence of $n-1$ larger values. Similarly, the D-S method has a problem when, for example, a proposition of mass of 0.9 repeated n times causing in the worst case assignment of 0.9^n , which can be quite small, when different sets are evidence are combined.

5.3 Level sets and mass assignment

A criticism of the approach in section 4, with respect to the D-S method, is that 1) no minimum and maximum likelihood value for the fuzzy result was provided and 2) no clear relationship between the two methods was shown. Recent work has addressed this issue by developing support pairs[16], generalisations of the Dempster-Shafer approach to fuzzy sets[19,17] and fuzzy interval analysis[18]. The development follows from Zadeh's possibility theory[20]. Some features of this work follows.

A fuzzy set can be represented in terms of its crisp α -level sets[18,21],

$$A_\alpha = \{ x \mid \mu_A(x) \geq \alpha \} \text{ such that} \\ \mu_A(x) = \sup \{ \alpha \in [0,1] \mid x \in A_\alpha \}.$$

Thus a fuzzy set is a nested family of level sets. When the family of a normalised fuzzy set (ie there is an x so that $\mu_A(x) = 1$) is finitely discrete then $\alpha_1 = 1 > \alpha_2 > \dots > \alpha_n$. Defining

$$m(A_{\alpha_i}) = \alpha_i - \alpha_{i+1} \text{ (with } \alpha_{n+1} = 0 \text{ by convention),}$$

the mass of the α -level set is the same as the Shafer's basic probability assignment, see section 3.

Also,

$$\mu_A(x) = \sum_i m(A_{\alpha_i}) \mid x \in A_{\alpha_i} \}$$

which is called the contour function by Shafer and is related [18] to the plausibility by

$$\mu_A(x) = Pl(\{x\}).$$

Figures 3 and 4 show the relationship of the fuzzy set A to its crisp α -level sets.

Thus a fuzzy set can be defined from the mass assignments but will not necessarily be normalised.

It is also true [19] that

$$A = \bigcup_\alpha \alpha A_\alpha,$$

where αA_α is a fuzzy subset whose membership function is $\alpha \mu_{A_\alpha}(x)$.

These representations based on level sets are used to make the connection between fuzzy set theory and the D-S theory evidential reasoning.

For the example of this paper, taking the ESM rf data as set A then $\mu_A(E1) = \sum_n m_{rf}(En) \mid E1 \in En \} = m_{rf}(E1) = 0.13$, etc. This is the assignment given in section 4 where $A=rf_range$.

5.4 Possibility Theory

In possibility theory, the possibility distribution function, r_A , associated with A is numerically equal to μ_A , ie $r_A(x) = \mu_A(x)$. This theory introduces the equivalent measures to support (also referred to as a belief function Bel [18]) and a credibility function Cr [10]) and plausibility which are certainty ($Cert$) (also referred to as necessity (Nec) [10,17]) and possibility ($Poss$) given by [19,20]

$$\begin{aligned} Poss[A/B] &= \sup_x \{ \mu_A(x) \wedge \mu_B(x) \}, \quad \wedge = \min, \text{ and} \\ Cert[A/B] &= 1 - Poss[A'/B]. \end{aligned}$$

Note that $Poss[A/B]$ is the possibility of A given B , both being subsets of X .

$$\begin{aligned} Poss[A/B] &= 1 \text{ if } A \cap B \neq \emptyset \text{ and} \\ Cert[A/B] &= 1 \text{ (if } B \subseteq A), = 0 \text{ (if } B \not\subseteq A). \end{aligned}$$

Thus [19]

$$\begin{aligned} Sp(A) &= \sum_j Cert[A/A_j] * m(A_j) \\ Pl(A) &= \sum_j Poss[A/A_j] * m(A_j). \end{aligned}$$

Therefore we can see that fuzzy reasoning is not in contradiction to D-S reasoning but rather a another method for dealing with uncertainty.

However, it is beyond the scope of this paper to review all aspects of these relationships covered here. An extensive review is given by Dubois and Prade [22,23].

5.5 Related applications

The example presented here is not the first application of these methods to the identification problem, see for example [5, 14, 15] for a more comprehensive case involving IFF data. Hickman[24] has looked at the problem in another ESM problem and had similar concern with independence of evidence, reassignment of mass away from the null set (normalisation), the combinatorial explosion and the lack of decision procedures.

6 CONCLUSION

The purpose of these notes was to show that, while Dempster-Shafer evidential reasoning can reach a useful conclusion for the identity attribute problem, it gets there with apparently unnecessarily tedious computation when compared with fuzzy reasoning. For consideration of additional emitters after the initial calculations, fuzzy reasoning can handle the situation using the previous calculations, which is not the case for D-S reasoning. As no apparent advantage is seen to be found from this computation, it is claimed that fuzzy reasoning is to be preferred when speed and simplicity is required for data fusion problems with uncertainty.

A discussion on the relationship of the Dempster-Shafer evidential reasoning method, based upon probability, to fuzzy set theory, via possibility theory, is presented to bring these methods to the attention of those readers requiring details.

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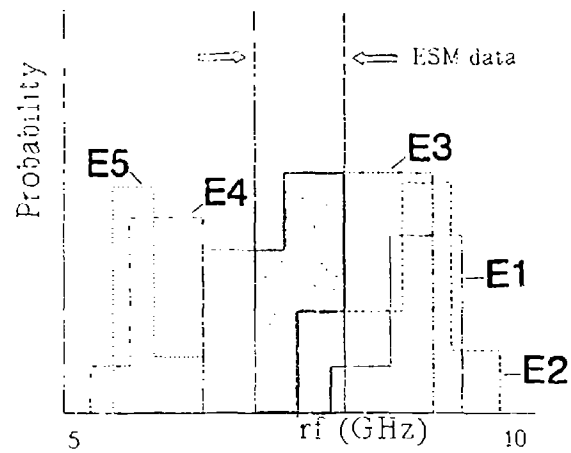


Figure 1. Emitter parameter distribution for carrier frequency. (Note the curves are only schematics.)

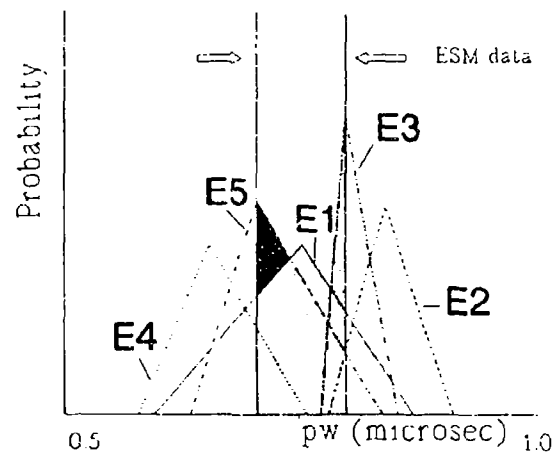


Figure 2. Emitter parameter distribution for pulse width. (Note the curves are only schematics.)

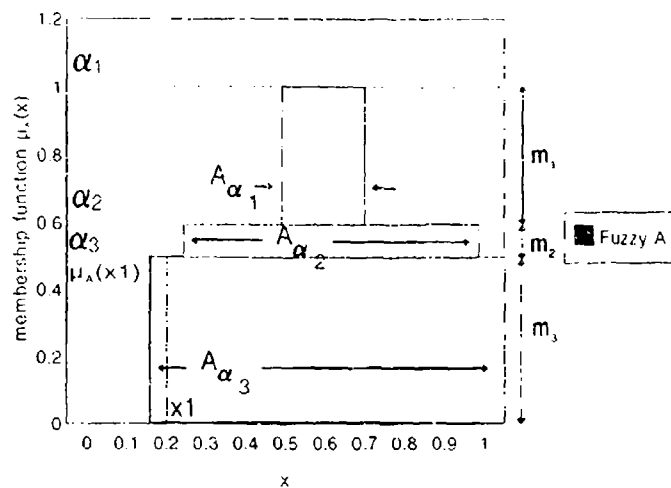


Figure 3. Fuzzy set A with its crisp α -level sets and evidential masses.

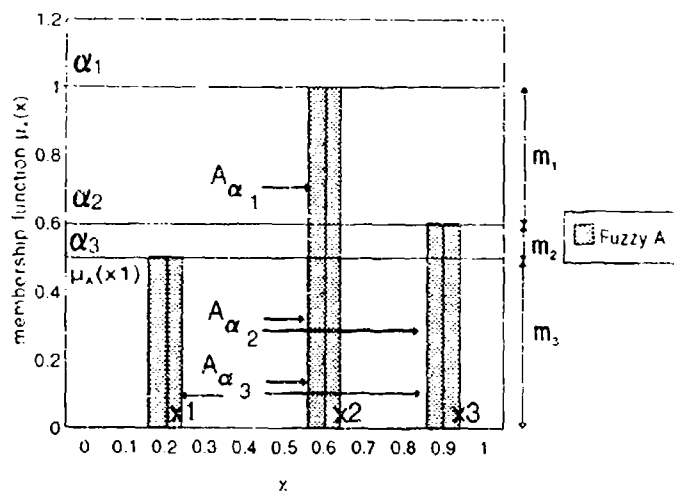


Figure 4. A discrete fuzzy set A with its crisp α -level sets and evidential masses.

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4. Title NOTES ON THE USE OF DEMPSTER-SHAFFER AND FUZZY REASONING TO FUSE IDENTITY ATTRIBUTE DATA		5. Security Classification <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">U</div> <div style="border: 1px solid black; padding: 2px;">U</div> <div style="border: 1px solid black; padding: 2px;">U</div> </div> Document Title Abstract	6. No. of Pages 22
		7. No. of Refs. 24	
8. Author(s) D.J. Kewley		9. Downgrading/Delimiting Instructions N/A	
10a. Corporate Author and Address Surveillance Research Laboratory PO Box 1500 SALISBURY SA 5108		11. Officer/Position responsible for: Security..... Downgrading..... Approval for Release..... OSRL.....	
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14. DEFTEST Descriptors	Fuzzy logic Artificial intelligence Data fusion Electronic support measures		15. DISCAT Subject Codes 120901, 1206, 1704
16. Abstract Dempster-Shafer evidential reasoning (D-S) has been applied to fusion of identity attribute data by Filippidis and Schapel. Their example involved deciding, from a given list, which radar emitter was producing the frequency and pulse rate measured by an ESM receiver. The D-S method was used to reach conclusions under uncertainty. Here an alternative approach using fuzzy reasoning is compared to their results. It is seen that the same conclusions are reached with a considerable reduction in computation. The ability to consider new data after a conclusion is reached is seen to be an additional reason for favouring the use of fuzzy reasoning for data fusion of this type. A discussion on the relationship between the approaches is provided.			

16. Abstract (CONT.)		
17. Imprint Surveillance Research Laboratory PO Box 1500 SALISBURY SA 5108		
18. Document Series and Number SRL-0094-TM	19. Cost Code 440/828304	20. Type of Report and Period Covered TECHNICAL MEMORANDUM
21. Computer Programs Used N/A		
22. Establishment File Reference(s) N/A		
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